Introduction to TerraMath/SedTec
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Content

Introduction ................................................................................................................................3
What is SedTec?...........................................................................................................................3
Project Targets ..........................................................................................................................3
Version Info ................................................................................................................................4
General Remarks ......................................................................................................................4
SedTec Framework ..................................................................................................................4
Data Model ...................................................................................................................................5
  SedTec supported data structure ..........................................................................................5
  Why was this structure chosen? .........................................................................................7
Functions handling cells and data set .......................................................................................9
Calculation Sequence ...............................................................................................................10
General Purpose Transport Functions ....................................................................................11
  Surface Transport .............................................................................................................12
  3D Transport .....................................................................................................................15
  Diffusion .............................................................................................................................17
Solid Dynamics .......................................................................................................................20
Implementation of Geological Processes ..................................................................................22
  General Remarks ...............................................................................................................22
  Scale & Simplifications .......................................................................................................22
  Processing Sequence .........................................................................................................23
  Environmental parameters ...................................................................................................23
  Tectonic ...............................................................................................................................24
Weathering and Regolith Production .......................................................................................24
Erosion, Transport & Sedimentation .........................................................................................25
  Erosion .................................................................................................................................26
  Transport ...............................................................................................................................28
  Transport distance ................................................................................................................28
  Modeling grain size distribution ..........................................................................................29
  Sedimentation processes ...................................................................................................29
  Consolidation .......................................................................................................................30
Introduction

What is SedTec?

SedTec is an abbreviation for Sediment and Tectonic. Initially SedTec was written as software to simulate erosion and sedimentation in dependency of various time dependent variables like tectonic subsidence and climatic change. As the first idea was the use for educational purposes, the intention was to start from the absolute beginning. It is an exceptional useful experience to encounter all aspects of geological numerical modeling starting with issues of the data structures, structure connected limitations, the processing pipeline (sequence of calculation) up to the simulated geological processes. Each of these steps is interacting and influencing each other and can not be separated.

Without consideration of the used data structure and associated methods it is not possible to answer the question if a certain software will or will not handle certain processes properly. Unfortunately software is most often used just as it is and users tend to be rather uncritical (especially if other researchers can be citied that used the software before) or just happy not to get too close to a very complex source code. Based on this experience SedTec was written with the idea in mind to offer to the researcher a framework of data handling methods – import/export, model generation, basic operations (geometry, cell management), model result validation and visualization. At the same time SedTec is not a monolithic block but can be extended by the demands of the researcher without a lot of work. In this way researcher can concentrate on their task (modeling of geological processes) but still taking influence in the implementation of methods which are crucial for their work. Included in SedTec from the beginning is a basic function set to model geological processes which may be used just as templates or if the results are satisfactory kept as they are.

Project Targets

The primary target of the project was software which is able to simulate sedimentary processes in a basin wide scale in dependency of climatic changes and vertical tectonic movements. A user defined number of grain sizes and number of sedimentary layers should be supported. Nowadays the framework aspect as described in the introduction is in the center of the development - the code should be as flexible as possible to cope with further not yet foreseen issues.
Version Info

At the time of writing (20080409) there are 2 versions of SedTec in use:

- Series 1.2x
- Series 4.x

Series 4 allows a higher degree of freedom during model calibration and has additional functions like:

- Mineral composition tables can be used to create models based on mineral distribution and not on source rock distribution
- Water is modeled in form of liquid water and ice, functions for ice melting / freezing included

Series 1 includes a history function (results of previous time steps can be displayed later on) and is easier to calibrate (but allows less user influence).

In the near future both software products will be combined to a new version 5.x which will support different license modes.

General Remarks

The target of this paper is to outline the ideas of SedTec in a clear and easy to understand way. Some text might sound trivial to the more experienced but my personal experience is that most problems of understanding start with simple things everybody is supposed to know (and never ask therefore). The text will be updated every time I have a free minute and I would be glad to receive (constructive) critics and ideas!

One intention to write this text is to make people conscious about the close link between data model / methods and the simulation itself – this might seem to be odd if at the same time I try to make a clear separation between framework and its application. The separation helps to keep the code easier to maintain and to adapt to new applications.

SedTec Framework

During the development of the software several tasks have to be solved:

Data Model – how is data stored
Interaction of data “units”
Implementation of (geological, physical, ...) processes
Handling of model boundaries
Data preparation
Visualization and analysis of results
Control of model results during simulation

<table>
<thead>
<tr>
<th>SedTec Objects</th>
<th>Process functions – data structure independent but not all structures might support all functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support functions</td>
<td>Import/Export</td>
</tr>
<tr>
<td>Higher level functions – data structure independent</td>
<td>Basic functions to handle data – data structure specific</td>
</tr>
<tr>
<td>Data structures</td>
<td></td>
</tr>
</tbody>
</table>

For all of these points several options are available and pro and contra has to be discussed.

**Data Model**

The questions are:
- How is data stored?
- How is good performance guaranteed?
- How to deal with memory issues?
- What influence does the data structure has / might have on future results?
- Internal structures of the smallest data units

Without considering the data structure and the methods connected with them, it is not possible to forecast what a model is able to deliver.

**SedTec supported data structure**

**Cell**

The smallest sub unit which is used within SedTec is called “cell”. Cells are geometrical homogenous volumes. Nevertheless if an internal structure is needed higher level functions are free to implement such features (e.g. thickness of weathered layer within a cell). The are implemented as field of floating point values with associated meta information, which makes it possible to interpret the stored data.
The cell stores status information and the material content. How the data stored in the cell is treated, is again the task of higher level functions.

The standard scheme is:

<table>
<thead>
<tr>
<th>Cell status information</th>
<th>Cell stored material</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 to 10 floating point values, includes values for: erosion, sedimentation balance, water content, flow / stored energy, temperature</td>
<td>Each substance is stored with ( n ) values, where ( n ) is the number of supported grain size classes</td>
</tr>
</tbody>
</table>

**Data set**

Whereas geometry is of limited importance to the content of the cell itself, it is very important if several cells are interacting.

In the most common case SedTec datasets are derived from grid datasets. This means \( x \) and \( y \) position of each cell is determined from the beginning and will not change during the whole simulation. In \( z \) direction cells have a higher degree of freedom and are not bound to a certain height or position. This is a necessity to be able to work with very thin strata in comparison to the extent in \( xy \) directions. The SedTec dataset is a 3 dimensional one with variable cell height.

The rigid structure in \( xy \) limits the usefulness for full 3d tectonic modeling as distortions is not native supported by the structure (simple vertical displacements are possible). In case tectonic movements in \( xy \) are large in comparison to the \( xy \) cell size use for this kind of modeling would be possible as well but is not the main focus if using this data structure.

SedTec is working with two data structures:

a) A low level 3d grid approach which permits high performance

b) A more advanced structure which is more memory optimized and permits insert into / delete at operations from the stratigraphic pile which might be useful for future extensions like plutonic / volcanic events. This data structure supports limited 3d tectonic modeling as described above.

c) (implementation in very early stage) Triangle / Tetraeder based network
Why was this structure chosen?

Cell structure

Before looking at the whole model considerations about the smallest unit ("cell") of the model have to be made:

• Is the cell rigid – or is deformation possible?
• Should be internal structures supported or not?

A cell which supports deformation has the advantage that processes like tectonic will be supported from the beginning – this is a huge advantage but leads to a considerable increase in coding time, decrease in calculation speed and supported model size as well.

As the focus of SedTec is sedimentary modeling of large areas the undeformable approach was chosen.

At first as mentioned before no cell internal structures were supported. This approach had to be altered to support processes like weathering to split a cell into a not weathered and a weathered part (the more flexible data structure b support cell splitting for example to support grain size sorting).

Data set

Grid approach

The most simple form to store data is in the shape of a sequence, giving some additional information like number of rows, columns, layers we can build 3 or higher dimensional objects without big problems. No other data structure will reach a better performance – simply because the computer has very little to do:

Accessing a certain cell in our model by world coordinates will look like (2d case, byte size of used variables ignored):

Index x = (Easting – ModelMinimumEasting) / cellsize;
Index y = (Northing – ModelMinimumNorthing) / cellsize;
Memory location = columns * index y + index x

The disadvantage is that often model areas do not form a rectangle – for example if we use drainage areas as input. In that case we will waste a lot of memory; luckily the data structures are rather small and even standard computers are nowadays 64bit machines with huge 16TByte (seen from a 2008 point of view) address space. A more severe limitation is that the minimum resolution must be already know when starting – smaller
features will not be included in a proper way (maybe leaving some influence when being averaged with other nearby features).

**Influence on results:**
If something should be transported from A to B there are just eight possibilities to go which is the number of the cells bordering cell A. No other directions than multiplies of 45 degree are possible. In simple models this can lead already to errors:

Assume a slope which directed 15°. In all movements the lowest neighbor will always be the one straight to north! Material running down the slope will head towards north not to 15°N as the difference to the northern cell is 15° to the NW cell 30°. Every transport will be directed to the wrong direction where the error is in the interval of 0 to less than 22.5°. In the next section functions will be discussed to minimize the resulting error.

**Triangle based approach**

Other data structures for example triangle based ones (TIN Triangulated Irregular Networks or in 3D tetraeder networks) are another approach. TINs are much more flexible than grids. It is possible to densify areas in regions where spatial variability is higher and to reduce the density in case no differences or small differences are detected. In that way it is possible to save memory and we are not bound to a “cellsize” (to a minimum resolution) – the resolution will be high where there is lot of data points and low where data is lacking. Moreover as the basic geometric element does not have a specific shape, deformation processes are principally supported.

**Influence on results:**
On the con side is the performance. It takes considerable efforts to find out which triangles / tetraeders are connected to each other. This is normally done by checking which triangles have how many vertices in common but even if using sophisticated algorithm (presorting in one dimension and combination with fast binary search) to limit calculation time, the speed decrease dramatically.

Another limitation is more severe as the quality of results is limited: Imagine a typical situation of a steep slope A (normally modeled with higher resolution) and a neighboring (lower positioned) plane B. Material will flow from A to B but where will the material deposited on B? If B is not subdivided dynamical into smaller triangles / tetraeders, the location of the sediments although coming from a geometrical well defined source location is not well defined anymore. The advantage of higher resolved areas is gone. Another disadvantage is that in a typical situation not only the topography defines areas where a higher resolution would be
advantageous but other input data (distribution of source rocks) as well. This leads to a higher memory consumption than a grid based approach.

This is why grid based structures were preferred in SedTec and approach C is not of high priority at the moment.

**Functions handling cells and data set**

The basic functions of the SedTec associated libs include:

**Cell functions**
- Cell initialization
- Set, get, add and multiply functions for single values inside a cell
- Set, get, add and multiply functions for a cell
- Calculating thickness, volume, density and mass of a cell
- Copy, move, delete and split of cells within the simulation data set

Most of these functions should not be used directly as this might lead to errors! E.g. according to definition the status data inside a cell includes values for porosity and or heat – simple multiplication will alter these values as well! Higher level functions sets have to implement their own functions adopting the low level ones.

**Neighborhood functions**
- Get topographic elevation field
- Get list of neighboring cells
- Get lowest / highest neighbor
- Sort list of cell by property
- Extrapolate property to nodata cell (see treatment of model boundaries)
- Calculating thickness, volume, density and mass of stratigraphic pile

**Data set functions**
- Init model
- Init time step
- End of time step behavior (see treatment of model boundaries)
- Calculation sequence

**Geometry functions**
- Vector operations
- Area inside / outside of polygon
- Cell A separated from Cell B (by line, plane)
- Slope

**Time table function**
- Interpolate values by use of a predefined timetable
• time interval overlap

**Calibration / Validation**

• Compare predefined value with cell / stratigraphic pile status at time \( t \)

**Visualization**

• 2D
• 3D
• Profile

All of these functions are not purpose specific and are the base for all higher level functions!

**Calculation Sequence**

Questions discussed in this section are:

1. Are results within a calculation sequence allowed to interact with other results or should they being kept separately? (and be put together at the end of the sub cycle)
2. At what position should calculation start?

Processes in nature are occurring more or less parallel with or without dependency to each other. As long as processes do not interact (spatial or temporal separation) there is no conflict.

Case 2 is of importance if intermediate results are allowed to interact with other parts of the model.

Example:
A snowball triggers an avalanche during the transport downhill material from other cells is taken away.

From experience it is easy to accept that this interaction should be possible but if yes the question number 2 is to solve:

Take a second look at the above mentioned avalanche. How will it develop if other processes have already taken away the downhill material, is it possible to neglect this? Of course not – if there is no nourishment for the snowball it will always stay a single snowball – in the best case.

The crucial question is to establish a ranking method which tells the system which event will happen first.
Possible approaches are:

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>Assumption that there is not enough information to establish a processing order.</td>
</tr>
<tr>
<td>Absolute value</td>
<td>Assumption the higher a certain value is the more easier some action is triggered.</td>
</tr>
<tr>
<td>1st Derivative (slope)</td>
<td>The bigger the differences to neighbor cells are the easier it is to start a process.</td>
</tr>
<tr>
<td>Value combination</td>
<td>A combination of above mentioned approaches, e.g.: Material will erode more easily if a) exposed (slope) and b) if the material has a low stability (absolute value)</td>
</tr>
</tbody>
</table>

Random mode can be used in case no rule can be established or to study how sensible the model reacts in comparison to the modes (blind test). Simply not to sort the data set would lead to the unwelcome effect that data structures (like rows, columns and their orientation) have an influence on the simulation result.

If the transported mass derived from one cell is not allowed to interact with other cells the sequence of processing is of no importance. But no trigger effects are possible.

**General Purpose Transport Functions**

Up to now two different transport schemes are distinguished within SedTec:

- Surface Transport – 2D
- 3D Transport

At this level of implementation we will not discuss why something is moving (e.g. gravity), how far and in what direction. Just if something has to be moved, how the process will be handled. Important is what material is transported: Is it possible to make subdivisions or is a single body simulated.

Another important issue is transport distance. As long as it is possible to manage that the transport speed is so slow that the distance (per time step) is less than the distance to the next cell, the number of affected cells is fixed and easy to manage. In the more general case such a limitation is not acceptable and a multi cell size distance transport has to be possible. Dependent on the implementation as shown later a very high number of cells might be affected; this will lead to the same situation as with limiting the transport distance per time step – in other words processing time will increase dramatically.
A typical situation where the transport distance problem can be shown easily:

Situation: Narrow valley with steep shoulders which are eroded. Material is arriving from 3 sides.

Case A: Transport distance limited to one cell size. Maximum mass which might be transported is limited to ½ of the elevation difference between to adjacent cells (else the topography might be inverted). Result: Other neighboring cells will move material into the channel before the first arriving material can leave. The system is very easily plugged, the channel filled up. To model incising valleys due to base level fall is not possible.

Analysis: The system is able to transport little or huge masses but speed of transport is constant. Like transporting goods with a small or big car at the same speed – the maximum distance in a certain time is the same just the mass will be higher using the big one. This is sometimes mixed up as in a standard life situation it is often expressed as: With car B the transport of the goods is x times faster (the throughput is higher but speed the same).

Case B: Transport distance not limited to one cell size. Maximum mass which might be transported is not limited to ½ of the elevation difference between to adjacent cells (but implemented so that the base of the source cell is not allowed to become lower that the elevation of the target cell prior to transport)

Result: Before other neighboring cells will move material into the channel, the target cell becomes source cell and is able to deliver material itself to a new target cell. How often this cycle is repeated depends on the actual material properties and situation (stored kinetic energy, viscosity, friction).

Analysis: The system is able to transport little or huge masses with variable situation depending speed. Plugging is still possible but avoidable. (The implementation in SedTec is named “Out of Order Processing”).

Surface Transport

Uniform movement

Situation:
- A mass A (stored within a cell structure) is moved on a surface S
- Start position P
- Vector of movement (v)
- Time step length t and dt (discrete interval after which the status will be updated)
If no previous movement is recorded, it is assumed that movement will start at the center of the surface cell. After each \( dt \) the position is updated. If no force is influencing \( A \), \( A \) will reach the position \( Q = P + t*v \). This is of course a very unlikely situation but even here some consequences have to be discussed:

**Position the moving mass \( A \)**

As a discrete xy grid is used to determine cell positions, position of \( A \) will normally not be equal to a cell center. If we simply use a nearest neighbor algorithm to determine the position, the error of the solution will be small if the cell size is small in comparison to the moved distance. Especially with slower motion – less than half of a cell size during time \( t \) – we will discover the problem that \( A \) is not moving at all – even during a high number of time steps where the cumulative distance is much larger than one cell size.

**Solution:**

a) In case that \( A \) can not be divided (into several packages which are distributed on all neighbors), the calculated position \( Q \) is stored and in the next time step this position together the updated moving vector will be used instead of a grid based cell center. At the end of the simulation \( A \) will be deposited at the nearest grid cell center.

b) It is possible to make subdivisions of mass \( A \), in this case \( A \) is distributed to all affected cells (on a grid based dataset most likely 4).

c) As already mentioned only multiplies of 45° will point without any error to the next grid cell. The vector of movement will be divided into a fraction that leads exactly to the center of the closest neighboring cell and a residual vector which will be used the next time the mass will be moved.

Solution c might be the most complicated but has a huge advantage – for the case of a non solid sub dividable mass – there is always just one grid location affected and errors are minimized by storing and later usage of the residual vector. Imagine solution b (more accurate) after each \( dt \) the mass will be distributed to 4 grid cells (which means because of different slope / azimuth of these cells) 4 different moving vectors and so on. A transport distance which is considerable larger than one cell size (but subdivided into smaller intermediary distances by using \( dt \)) will lead to \( 4^{(\text{transport distance} / \text{cell size})} \) affected cells which leads to a dramatic increase of calculation time!
Accelerated movement

Normally the movement of mass A will be influenced by other forces as well:

- Gravity
- Friction (derived from material properties of surface and moved)

The direction of friction will be directed against the moving direction of A and just influencing the length of the moving vector of A. But other force vectors will normally have a different orientation.

At the present state of discussion the calculation of the influence of gravity or friction is of no interest – important is just that we have a possibility to influence the movement at a certain time with a certain component.

Knowing the forces on A the moving vector $v$ and the position will be updated every $dt$. 
3D Transport

Status:
3D Transport implementation is not fully tested yet (20080409)!

Definition:
3D Transport is a movement of mass or energy inside the simulation dataset without any restriction. Instead of vectors like for the surface transport scalar values are used.
Processes which might use this kind of transport are:

- Sorting by density / Sedimentation
- Heat flow
- Water flow (pressure differences)

The target of 3D Transport function is to minimize potential differences inside the model. Again in this section it will be only discussed what is done to achieve this target, not what the cause of differences or the speed of transport might be!

The biggest difference to the above mentioned Surface Transport is that the movement should have a higher degree of freedom which means that all neighboring cells have to be checked.

In case of a voxel model which would have constant distances in all three dimensions of space, it would be quite simple as all neighbors are already known. As the SedTec data model permits a higher degree of freedom in z direction, more steps are necessary:

- Get a list of all neighboring cells
- Calculate the contact surface area of each neighboring cell which is exposed to the central cell (the cell where calculation is started) – cells with volume = 0 are ignored

The size of the contact area and the local difference to a neighbor define together with material properties (heat conductivity, viscosity and so on) the activity index of exchange / transport.

- Rank cells according to absolute difference multiplied with conductivity
- Handle all listed cells and update the central value after each calculation

Again – like in the 2D case – there is the problem distributing to more far cells. If all are handled the number of needed calculation will even increase with power 3, possible solutions:
a) Limit transport to nearest neighborhood – this will lead huge errors as every transport has same speed
b) Calculate all affected cells down to a boundary value (e.g. difference smaller than 1% of original difference)
c) Just proceed to the neighbor with the highest difference – this will lead to errors as well as not all cells have same size:

1D example (possibility c):
A cell A is bordered by 2 others: B & C
B has a larger contact surface but smaller a smaller difference, C a very small contact surface but a high difference. If the product of contact surface and difference is used transport will always go just to B never to C (and the huge value contrast will never be reduced). If just the value difference is used, it is not possible to transport high values because of small contact surface. Especially if the contact surface / volume of C is close to 0, a more or less static situation is created.

Solution b is at the moment the favored one!
**Diffusion**

The last paragraph focused on transport processes seen more from the programmer’s point of view. From this point of view important issues were things like:

- Performance – how many cells have to be processed
- How to decide which objects inside the model have to be processed
- What functions have to be created so that the actual transport process can be implemented

The next step is to combine the prepared functions and introduce a theoretical concept.

As mentioned before it is impossible to treat all particles within a sediment model on an individual base, reasoning:

1) calculation devices do not have enough resources
2) there is not enough information about the single components available

If we take a look on general diffusion equation of the form:

\[
\frac{\partial}{\partial t}c(\vec{x}, t) = D \nabla^2 c(\vec{x}, t)
\]

where D might be interpreted as diffusion coefficient or maybe thermal conductivity and c as concentration or temperature (just to mention two possibilities), we have a multi purpose tool for modeling.

The advantage here is that not single particles are treated but the statistics of randomly moving. Of course random here does not mean without a cause but that one movement can not linked with another (due to lack of information). Inside the mass of particles no exact movement will be calculated. The diffusion equation is not able to do so but can predict the sum of movements of a population of particles which share similar statistical properties.

Diffusion takes place in environments where viscous behavior is dominating which is the case for very low Reynolds Numbers.
\[ R = \frac{\rho v_s r}{\eta} \]  

\[ v_s \text{ Characteristic speed of the particle} \]

where \( v_s \) might be expressed as:

\[ \langle v_s \rangle = \sqrt{\frac{k_B T}{m}} \]  

\[ v \text{ Average speed} \]
\[ \sigma \text{ Proportionality factor} \]
\[ F \text{ Applied force} \]

The difference can be explained by the observation that energy (the applied force) is very fast converted to other forms of energy (thermal).

\[ \gamma = 6\pi \eta r \]

Velocity in a viscous system is therefore proportional to the applied force.

This would imply that all movement is immediately stopped as soon as the driving force cease. Observation shows that this is a valid assumption for viscous flows but not necessarily true for all mass movements:

- Rock falls or avalanches, where components are accelerated so much that they move up on the opposite slope
- Water moving on a horizontal plane after running down an inclined channel

This does not mean that the approach to use diffusion is incorrect but that there are cases where we have to consider other approaches as well!

For a more detailed introduction to the diffusion equitation see: Ursell, 2007.
As shown in this paper deduction of other properties important for modeling sediment flow is possible:

The mobility is inverse to increasing grain size and viscosity (using the inverse of Stokes drag).

\[ \sigma = \frac{1}{6\pi \eta r} \]

To address the already mentioned short comings of a pure diffusion type implementation, principles of classical mechanic were implemented as well:
Solid Dynamics

To treat a moving mass as high viscous medium is acceptable in some but not all situations and depends on the resolution of the modeled system. Mass inertia - not treated in the diffusion system - plays a significant role in situations where the moving mass behaves like a solid body system. This includes mass movements like rock fall but altering stream courses as well.

As a mass (cell) is treated as one singular body (although consisting of a nearly infinite number of particles), usage of principles / formulas of solid mechanics is possible:

- Newton’s axioms (laws of inertia, acceleration & reciprocal actions)
- Friction (sliding & rolling) - dry friction, Stokes drag

All formulas are used in vector form. Frictional forces are directed against the direction of movement. The gravity vector is always directed to the location of the lowest neighbor. The kinetic energy vector is the resulting vector of all previous movements.

\[ E = E_{\text{kin}} + E_{\text{pot}} - E_R - E_I \]

- \( E_{\text{kin}} \): Stored kinetic energy
- \( E_{\text{pot}} \): Potential energy which is converted into kinetic during the actual time step
- \( E_R \): Surface friction (\( F_R = \mu F_N \)) & Stoke’s drag
  \[ F_d = -bv \]
- \( E_I \): Internal friction, \( F_I = A\eta dv/dz \)

Abbreviations:
- \( m \): mass
- \( g \): gravity constant
- \( v \): speed
- \( h \): elevation difference between source and target cell
- \( \mu \): friction coefficient
- \( F_N \): normal force
- \( A \): area of cell
- \( \eta \): Viscosity
- \( z \): thickness of cell

The driving force of the transport is gravity: \( F_g = mgh \)
Whereas diffusion equitation is used to describe 3d flow, solid dynamics is focused more on surface transport. For estimating the mass which should be transported an approach similar to diffusion modeling is used!
Implementation of Geological Processes

General Remarks

In this section all processes are discussed which focus on use in sediment modeling. The functions are based on the SedTec framework described in the previous section. At the present state of development some functions which are a specialized case of a more general solution are still implemented as independent function – these cases will be removed in the future to simplify the code management. As already mentioned in the previous section, there are always several possibilities for implementation and it depends on the skills, resources and focus of the work what and how features are implemented.

Scale & Simplifications

If the task is to calculate the path of a ball moving down an inclined surface, it is possible to solve the task with a very high spatial and thematic resolution with today hard- and software. Even if the number of balls (mostly called boulders then) is increased to several hundreds or thousands, it is still possible to simulate real time models. And as this most often used for processes that happen in a comparable time we do not have to worry about time scale problems as well too.

Spatial
If a large area (sedimentary basin and source area) is considered, there are so many particles that it is senseless and impossible to model every single grain. The approach chosen in SedTec is to model the flow of sediments as viscose liquid. Therefore to model grain size distribution grain size classes are used. Spatial resolution is most often in the range of about 100m cell size (xy, z-axis has no limitation). Sedimentary features of smaller size will simply not being resolved.

Time
If time steps have the duration of many thousands or millions of years even a sub cycle will have durations of hundreds or thousands of years. In this time a cumulative precipitation of hundreds of meters may fall on the model. This means the amount of water has to be scaled down to a level where rivers and lakes have a realistic dimension. If the amount of water is scaled down, the effects of the water are scaled down as well. So the properties of water have to be changed in the way that although very little mass it will have the same/similar effects as a water flow built up by much more water.
Water itself may have a huge influence on the morphology (glaciers during ice age) and transport speed of sediments (altering their “viscosity”).

**Processing Sequence**

A SedTec Processing sequence is build up by following calculation steps:

<table>
<thead>
<tr>
<th>Data &amp; parameter import</th>
<th>Model settings, time tables (environment, tectonic), Tectonic geometry (subsidence areas), source points, control points</th>
</tr>
</thead>
</table>
| Model Initialization    | - Extraction of litho ids from supplied lithological map  
                        | - Initialization of data structure                                                                                     |
| Time step 1 – n, cycle of: | - Update of environmental parameters  
                        | - Tectonic  
                        | - Weathering  
                        | - Erosion  
                        | - Transport  
                        | - Sedimentation  
                        | - Consolidation  
                        | - Update report (status of control points, average flow, transport distance, erosion, ..) |
| End of simulation       | Export of model or extraction of parts                                                                               |

In nature all processing steps inside a time step (reporting excluded) will be parallel in time, using computers a semi (pseudo) parallel computation may be achieved using a very small stepping (increasing the number of calculation steps).

**Environmental parameters**

Time dependent environmental parameters are updated according to the actual time using a supplied timetable (the time table is mandatory).

Time dependent parameters are:
- Sea level
- Temperature
- Precipitation
- Sediment supply

Values are interpolated linear. Precipitation is variable in time and space if a precipitation map is imported (the spatial distribution itself is constant in time).
Series SedTec 4.x allows finer time depend tuning exposing more parameters inside the time table and global settings. This includes

Climatic variations between warmer and cold periods are modeled using a variable precipitation and sediment supply rate. Increased precipitation leads to higher erosion and to a reduced viscosity of the transported sediments. The sediment supply mimics climatic and vegetation variations that will lead to a higher or lower erodiability (rock shattering by frost or increased temperature variation, changes in vegetation coverage).

**Tectonic**

Tectonic in SedTec is a passive input parameter. In other words the user defines areas or blocks by polygons which are associated to a time history file. Within a history file a sequence of entries define which area is when active and the style of movement (simple translation in z or with a rotational component). Faults are assumed to be rather steep and the horizontal movement vector in comparison to the cell size is neglect able.

**Weathering and Regolith Production**

Physical weathering is modeled as an in situ process which leads to a reduction in grain size and increase of erodability, implemented rules are:

- a) Exposition – how much of a cell is exposed to the surface, the more the higher the weathering
- b) Orientation of exposition towards an user defined sun position
- c) Temperature, weathering is positive correlated with temperature
- d) Temperature variation – the higher the variation the higher the weathering
- e) If temperature cross the 0°C mark frost shattering increase the weathering
- f) Vegetation coverage (strongly simplified model)

<table>
<thead>
<tr>
<th>Abbrevation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{veg}}$</td>
<td>Vegetation factor which is derived from a fuzzy rule set where the influence of temperature, slope and precipitation is weighted</td>
</tr>
<tr>
<td>$f_{\text{exp}}$</td>
<td>Exposition factor describes the exposed surface of a cell, top and all free sides of the cells are added and divided by the top surface</td>
</tr>
<tr>
<td>$f_{\text{fac}}$</td>
<td>Weathering temperature activity, see formula</td>
</tr>
<tr>
<td>$f_{\text{fs}}$</td>
<td>Frost shattering factor: dependent on the temperature interval, if the 0°C line is crossed, the weathering is increased by an user defined constant value. How often this happens is estimated from the mean temperature and its fluctuation. $f_{\text{time}} = \text{fabs}(\min_t)/\text{fabs}(\max_t)$ if ($f_{\text{time}} &gt; 1$) $f_{\text{time}} = 1/ f_{\text{time}}$</td>
</tr>
<tr>
<td>Process</td>
<td>Equation</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>Weathering, temperature activity</td>
<td>$f_{ac} = t_k^{\cdot p} / t_{273.15}^{\cdot p}$</td>
</tr>
<tr>
<td>Weathering</td>
<td>$f_w = f_{veg} \cdot f_{exp} \cdot f_{ac} \cdot f_{fs}$</td>
</tr>
</tbody>
</table>

**Calculation of weathering depth**

$$\frac{\delta R}{\delta t} = W - \Delta Q_s$$

- $R$: regolith depth
- $W$: regolith production rate
- $Q$: Surface material flow

Weathering depth will not exceed a maximum value (user defined) for a just top surface exposed cell at a certain temperature. If the maximum weathering depth is not reached yet (or material got lost by erosion) the weathering depth will be increased by a user defined speed ($W$). If no more weathered material available erosion does not stop but it is slowed down as the erodability of fresh material is lower.

**Erosion, Transport & Sedimentation**

The simplest assumption to model sediment flow is a diffusion formula like:

$$Q_x = -k \frac{\delta z}{\delta x}$$  \(Q_x\) material flow, \(k\) transport efficiency

which results in a slope dependent flow (modified versions e.g. Carson & Kirby, 1972). To enable channel incision stream power terms were added which take in consideration the erosion power of the mass flow. This power is estimated by the local slope and the size of the drainage area:

$$\frac{\delta z}{\delta t} = dSA$$  \(d\ldots\) coefficient of erosion, \(S\ldots\) local slope, \(A\ldots\) drainage area size; e.g. Hancock et al. 1998

Combination of these approaches lead to algorithm using slope and transported mass over a distinct location (Kirkby, 1980, Dunne and Aubry, 1986) to estimate erosion.

Models which use flow rate to calculate how much material can be transported to lower neighbors instead of just using the slope, have the advantage that the movement will not immediately stop when the stream enters a plane (if slope and flow are additive terms and not multiplied). The flow is reduced as the flow channel widens and the material is distributed on a higher number of cells. Flow
reduction in this case is a pure geometrical effect. But in channels of constant width with declining slope acceleration will still occur until the slope is zero and the mass will move on in state of uniform movement. Slow down of movement can be reached by using geometrical considerations (Target is not allowed to be higher than source after transport). As this is rather unlikely and a pure geometrical correction leads back to just slope controlled which cause undesired effects especially in low slope narrow valleys (plugging). SedTec works with formulas of mechanical physics to give a more complete image of the energy cycle and allows multi cell transport distances to avoid geometric inconsistencies.

**Erosion**

Amount of erosion is estimated by using an erodability factor which defines the reduction of elevation difference between source and target cell within a certain time interval. The erodability factor can be different for each used bedrock lithology. As soon as the material is in the state of transport and consists of loose particles of different origin and size, the bedrock factor can not be longer applied and a viscosity factor based on water content and particle size is used to calculate the amount of material that will be moved to the next target cell.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_{surf}</td>
<td>Default value 1</td>
</tr>
<tr>
<td></td>
<td>If z &lt; sea level and z &gt; sea level – wave base a user defined value is assigned to increase the erosion within the surf zone (value variable in time)</td>
</tr>
<tr>
<td>f_{landsea}</td>
<td>Default value 1</td>
</tr>
<tr>
<td></td>
<td>If z &lt; sea level a user defined value is assigned (time invariant)</td>
</tr>
<tr>
<td>f_{weath}</td>
<td>The higher the value the more regolith will be produced. Regolith needs less energy to be eroded than solid bedrock.</td>
</tr>
<tr>
<td>f_{env}</td>
<td>Composite factor; f_{env} = f_{weath} f_{landsea} f_{surf}</td>
</tr>
<tr>
<td>d_{bedrock_rodability}</td>
<td>Material specific constant for a certain bedrock lithology, set in the material table.</td>
</tr>
<tr>
<td>dz</td>
<td>Surface elevation difference between source and target position. The higher the difference the more energy is available for erosion.</td>
</tr>
<tr>
<td>d_{dry_regolith}</td>
<td>assumed diffusion factor of a loose not compacted sediment</td>
</tr>
<tr>
<td>d_{water}</td>
<td>Diffusion factor of water</td>
</tr>
<tr>
<td>c_{water}</td>
<td>Water content [0;1]</td>
</tr>
<tr>
<td>p_{consolidation}</td>
<td>Consolidation is pressure dependent in SedTec, it is assumed that at a certain pressure sediment reach the viscosity of the bedrock components again. Before this pressure is reached the landscape diffusion factor is scaled linear between bedrock diffusion factor and diffusion factor of dry loose sediment under consideration of water content.</td>
</tr>
<tr>
<td>p_{actual}</td>
<td>Actual average pressure on a cell: Equal to height of overburden multiplied by its density plus 0.5 of the cell weight.</td>
</tr>
</tbody>
</table>

Tab. Xx : Erosion & transport processes guiding parameters

| Diffusion factor of sediments | p_{ratio} = p_{actual} / p_{consolidation} |
\[ dd = d_{\text{bedrock
erodability}} - ((1 - c_{\text{water}})d_{\text{dry
golith}} + c_{\text{water}})d_{\text{water}} \]
\[ d_{\text{actual}} = f_{\text{env}}(d_{\text{dry
golith}} + p_{\text{ratio}} \cdot dd) \]

Tab. XX: Formula of diffusion factor of sediments

<table>
<thead>
<tr>
<th>Equation</th>
<th>Abbreviation</th>
<th>Energy condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic energy ( E_{\text{kin}} = \frac{1}{2}mv^2 )</td>
<td>( m ) … mass</td>
<td>( E = E_{\text{kin}} + E_{\text{pot}} - E_{R} - E_{I} )</td>
</tr>
<tr>
<td>Potential energy ( E_{\text{pot}} = mgh )</td>
<td>( g ) … gravity constant</td>
<td>( E_{R} = F_{R} \cdot \text{dist} )</td>
</tr>
<tr>
<td>Surface (sliding) friction force ( F_{R} = \mu \cdot F_{N} ) ((F_{N} = mg))</td>
<td>( h ) … elevation difference between source and target cell</td>
<td>( E_{I} = F_{I} \cdot \text{dist} )</td>
</tr>
<tr>
<td>Internal friction ( F_{I} = A\eta v/dz )</td>
<td>( v ) … speed</td>
<td>dist = distance between center of source and center of target cell</td>
</tr>
<tr>
<td>Energy equitation of a mass in state of transport</td>
<td>( \mu ) … friction coefficient (constant preset or estimated by roughness of neighborhood matrix)</td>
<td>( E_{R} ) and ( E_{I} ) are always directed against the direction of transport</td>
</tr>
<tr>
<td></td>
<td>( F_{N} ) … normal force</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( A ) … area of cell</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \eta ) … viscosity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( z ) … thickness of cell</td>
<td></td>
</tr>
</tbody>
</table>

Tab. XX: Erosion & Transport formula

For calibration purposes the components of the energy equation can be scaled using a formula of the form:

\[ f(x) = a \cdot x^b \]

Scaling is a necessity as the software simulates by simplified formulas and empirical values and allows using it on different environmental conditions.

The algorithm used in SedTec

\[ E = E_{\text{kin}} + E_{\text{pot}} - E_{R} - E_{I} \]

for estimation of energy which is available for erosion and transport plus the situation dependent material diffusion factor (bedrock, loose sediment, water content) enables to simulate different situations. Erosional and transport processes in SedTec are not divided in hillslope and bedrock channel incision like in other landscape evolution models (Densmore, 1998). Typical consequences of used formula on model are:

<table>
<thead>
<tr>
<th>Examples</th>
<th>Assumptions</th>
<th>Energy condition</th>
<th>SedTec response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hill slope</td>
<td>• low amount of water per area&lt;br&gt;• flow covers whole</td>
<td>• equal transport over the area – uniform erosive&lt;br&gt;• no or rare distinct chan-</td>
<td></td>
</tr>
<tr>
<td>Slope/Region</td>
<td>E\text{\textsubscript{kin}}</td>
<td>E\text{\textsubscript{pot}}</td>
<td>E\text{\textsubscript{R}}</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>Channel</td>
<td>increasing downstream</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>Fan surface</td>
<td>decreasing downstream</td>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>Plane</td>
<td>very low</td>
<td>low</td>
<td>const.</td>
</tr>
<tr>
<td></td>
<td>decreasing</td>
<td>low</td>
<td>const.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Transport**

It is assumed that while transporting the eroded mass downstream the potential energy \( E_{pot} = mgh \) is transformed to kinetic energy \( E_{kin} \). Additional to the newly transformed kinetic energy there might be an additional \( E_{kin} \) stored in the cell generated by previous transport. During the transport a certain part of the energy is used for friction work - internal friction (work between the particles of the moved sediment) and friction between the surface and the moved sediment (in other words for erosion).

**Transport distance**

SedTec assumes that a certain part of the kinetic energy of the moving mass is consumed by two processes, the friction of the moving mass against the surface and the friction between its particles (Tab. XX: Erosion & Transport formula ).
As long as the slopes exceeds a certain value the speed of the mass will increase and will be decreased if more energy is consumed than generated by the elevation difference. How much energy is needed depends on the composition of the moved volume – especially on the water content.

**Modeling grain size distribution**

Storing actual grain sizes for all particles is not possible. To be able to give information about grain size development during transport a different approach was used:

All particles were assigned to certain grain size classes and for each material a certain reduction factor by unit distance is defined. When being transported a certain fraction defined by the grain size reduction factor will be moved to the next smaller class. Using this approach it is possible to simulate the reduction of grain sizes with distances but also to simulate depending on substance properties.

**Sedimentation processes**

Following parameters are used to simulate the deposition of particles:

- Calculated viscosity, defines how much time is needed for deposition
- Density contrast between the average density of the cell and a single grain size class of a substance
- Estimated flow speed
- Particle mass

The range of viscosity is between the viscosity of water and the viscosity of a waterless sediment pile (\(v = 1/d\)). The flow speed is estimated from the transport distance and the time interval for one calculation step (not equal to time step duration). Particle mass is calculated from the average grain size of the grain size class and the material depend density.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average cell density</td>
<td>(d_{\text{cell}} = \frac{\sum m_i}{\text{vol}})</td>
<td>mass of a certain grain size of a certain material; vol … volumen of a cell</td>
</tr>
<tr>
<td>Particle mass</td>
<td>(v_{\text{par}} = \frac{r^3 \pi}{3})</td>
<td>It is assumed that particles have a sphere shape</td>
</tr>
<tr>
<td>Deposition factor</td>
<td>(f_{\text{depos}} = \frac{v_{\text{par}} \times d_{\text{diff}}}{(\text{viscosity} \times v_{\text{flow}})})</td>
<td>Deposition increase by the density contrast between a single particle and the average density of the cell, the size of the particles and decrease by viscosity and flow speed.</td>
</tr>
</tbody>
</table>

Tab. XX: Calculation of material and grain size depending deposition factor.
If transport distance modeling is used only transport distance is used to model the grain size distribution, sedimentary processes are switched off – this mode was used to generate the models presented within this paper!

**Consolidation**

Consolidation is defined as the process of decreasing the diffusion factor by pressure of overburden and reduction of the cells water content. Reduction of water leads to compaction.

Consolidation of sediments using following assumptions:
- The higher the water content the higher the diffusion factor
- Pressure increase the consolidation

The calculation follows the formula for diffusion factor of sediments (Tab. XX). Water is reduced by fixed value by pressure, composition of overburden is neglected.

If a certain pressure is reached, the material will be again defined by the original bedrock properties. Cementation processes are not implemented.